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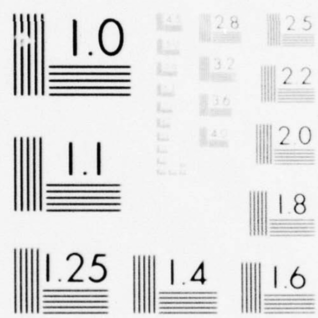
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Research Report CCS 299

# STRATEGY SELECTION IN ORDINAL RANKING PROBLEMS

by

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ABSTRACT

This paper investigates a class of committee ranking problems. A given member wishes to select an ordinal ranking of a set of objects which, when combined with the rankings provided by the other members, yields a consensus ranking that is as close as possible to some desired set of preferences. Areas of potential application such as defense construction and highway planning are discussed. An algorithm for determining the best ranking together with an illustrative example is given.

## 1. Introduction

This paper investigates ranking problems in which each member of a committee provides an ordinal ranking of a set of objects or projects, and in which a given member of the committee wishes to have the resulting consensus be as close as possible to some desired ranking. Specifically, consider the problem in which $m$ committee members have given ordinal rankings to $n$ objects. These rankings are denoted $\{Q^\ell\}_{\ell=1}^m$, where $Q^\ell = (q_1^\ell, \ldots, q_n^\ell)$ and $q_j^\ell = i$ means that the $\ell^{th}$ member has given object $j$ rank $i$. The consensus ordinal ranking is given by ordering the sums[1]

$$\sum_{\ell=1}^{m} q_j^\ell + x_j, \quad j = 1, \ldots, n,$$

where $X = (x_1, \ldots, x_n)$ is the complete ordinal ranking of the last committee member. This final ($m+1^{st}$) member has a desired ranking $G$. We assume, without loss of generality, that $G = (1, 2, \ldots, n)$; that is, the natural order is the desired one. The problem is to find that ranking which makes the consensus ranking as close to $G$ as possible. By closest we mean the ranking for which the most ordinal relationships from $G$ hold.

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[1]Although there are numerous methods of formulating a consensus, we have adopted the most straightforward, namely, Kendall's average. See [8].

The model as defined requires that $y_j = \sum_{\ell=1}^{m} q_j^{\ell}$ be a known quantity to member $m+1$. Although this is a restrictive assumption in many cases, it may in fact be the only point of departure from which to launch an investigation of strategic planning. Other approaches such as a maximin criterion would not only be computationally unwieldy, but would need to be based on erroneous assumptions and would lead to useless results. Such an approach would generally mean assuming (i) that the first $m$ members know member $m+1$'s desired ranking; and (ii) that they would be forming a coalition to enter into direct competition with member $m+1$. In most applications these assumptions are unfounded.

The approach taken in this paper leads to an easily implemented algorithm which determines the best $X = (x_1, \ldots, x_n)$ for any $Y = (y_1, \ldots, y_n)$. Several directions for a more detailed analysis are then possible. One such direction is to investigate the nature of $X$ for several $Y$ vectors which the decision maker might feel form the boundary of possible variation in the preferences of the other $m$ members. Given this set of $X$ vectors, an acceptable one could, in most instances, then be selected as the "best."

An alternate direction is to initiate a form of sensitivity analysis. For a given computed $X$, pairwise variations in the $y_j$ components can be investigated to determine the ranges over which $X$ remains optimal. Such an analysis for various $X$ can provide substantial information as to which strategy (ranking) member $m+1$ should select.

In the following section we discuss a number of areas of application of the strategy selection problem. In Section 3, we present an algorithm

for solving the strategy selection problem, and discuss the problem of weights. Section 4 contains an illustrative example.

## 2. Applications

This ordinal ranking problem finds application in a number of areas. Consider, for example, the problem of assigning priorities to defence construction projects. For the Canadian military, one of the first steps in the decision process is to establish the relative importance to be attached to various areas. That is, proposals will be submitted for projects in the areas: maintenance to base buildings, recreation facilities, road and parking lot repair, lighting facilities in hangars, defence educational facilities, . . . , etc. When representatives from the various military bases throughout the country gather to establish (as a group) what emphasis should be placed on the various categories, in a preference ordering sense, each representative has a different ranking in mind. One base may have its greatest needs in the area of operational and security facilities, while another may wish to emphasize defence education and training facilities.

Consequently, each representative ranks the categories in order of relative importance in relation to the perceived needs of the military base in question. This ranking is done on an individual basis initially and then the average ranking computed by a referee. A general round-the-table discussion then follows in order to get the views of each member present. The members then rank the categories again, and an average is recomputed. In this way, the averages tend to converge to a ranking which is not too dissimilar from the desires of the representatives present.

For more detailed information on the process of project funding and priority assignment in defence construction see [1], [4] and [13].

Another area of application arises in highway planning. As a case in point, consider the problem recently faced by a consulting firm assigned the task of selecting a corridor (a 500-ft. wide strip of land) through which a 4-lane highway would be constructed.[2] This highway was being built in order to bypass the city of Peterborough, Ontario. The procedure followed by the firm in this instance, (and which would generally be the procedure followed in any corridor selection problem) was to carry out a public opinion poll in each municipality concerned. A group of municipal officials (municipal body) was responsible for conveying the wishes of the people from that municipality to the consulting firm.

A sample of residents from each municipality was selected by the officials and each resident asked to specify how a corridor passing through their municipality would affect various things. Factors considered were: (1) regional development benefits such as employment opportunities, increase in business profits, net change in housing costs; (2) user benefits such as change in travel time, operating costs, accident costs, user comfort due to changes in geometric standards . . . etc; (3) social benefit such as employment stability, community growth, community cohesion; (4) environmental benefits such as noise, air and water pollution, loss of natural land areas, damage to fish and game, damage to farm land, . . . , etc.

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[2]The authors were indirectly associated with the Peterborough Bypass study.

The consulting firm then presented each municipality (municipal body) with a series of "possible" corridors. Some of these corridors would, if constructed, pass through a given municipality, while others would not. Given the opinions gathered from the sample and relating to the above factors, the municipal body then constructed a preference ordering of the corridors from "best" to "worse." This ranking was designed to reflect residents' opinions, as well as the groups' assessment, of the long-run benefits which the highway would yield.

The firm then collected the rankings from the groups, and attempted to construct a representative compromise. Essentially, the compromise ranking amounted to a "weighted" average. Weights were assigned on the basis of population of the municipality and the total benefit/damage factor for the municipality. From this consensus some of the proposed corridors were dropped, the remaining ones specified in finer detail, and this subset resubmitted to the municipalities for a second round of preference specification.

This procedure was carried, at later stages, to "higher" levels of authority (than just the municipal) until a final "best" corridor was decided upon.

For further details on the above-mentioned study and related work see [5], [7], [10], [15].

In both of the above problems there is an inherent strategic aspect which is worthy of investigation. In the corridor selection problem, for example, a given municipal group may view the process as one where it must select the most effective ranking $X$ of the options. As in many competitive decision making frameworks, the member (player) under

consideration must either assume, for each opponent, a pure strategy ($Q^{\ell}$) or else a mixed strategy or prior distribution over the possible rankings. This is the approach taken in many competitive bidding ([3], [9], [14]) and advertising ([2], [11], [12]) models.

The desired ranking $G$ would be developed based on what the municipality perceives as being the benefit(damage) implications of the various factors being considered; that is, wildlife, forest, business, . . . , etc. The problem facing that municipality is, therefore, to select a ranking $X$ which, when combined with the other $n$ rankings $Q^{\ell}$, will yield a ranking which is as close as possible to $G$.

By a similar argument the defence construction problem exhibits an element of strategic planning. A given base, in wishing to reach a consensus which is consistent with its interests, would want to select the most appropriate ranking. Again, the matter of weighting may enter the picture since factors such as total dollar volume requested and size of base must be considered.

Other problem areas such as water resource planning and highway maintenance budget allocation exhibit a strategic ranking element worthy of consideration. We will not elaborate here.

## 3. Solution Procedure

In this section we present a branch-and-bound algorithm for solving the strategy selection problem. We begin by giving the following mathematical programming representation.

$$\text{Minimize} \quad \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} z_{ij},$$

subject to

$$y_i + x_i \le y_j + x_j + Mz_{ij}, \quad 1 \le i < j \le n, \qquad (3.1)$$

$$z_{ij} \in \{0, 1\}$$

$$X = (x_1, x_2, \ldots, x_n) \in C$$

where $M$ is a large positive number and $C$ is the set of all complete rankings.

At optimality the objective value of (3.1) will specify the number of ordinal relationships from $G$ which cannot be satisfied. The zero-one variable $z_{ij}$ equals one when the optimal strategy $\overline{X}$ does not yield a consensus ranking which satisfies the desired ordinal relationship between projects $i$ and $j$. Of course, alternate optimal strategies are possible.

To simplify notation, we define $d_{ij} = y_j - y_i$ and rearrange variables to obtain:

$$\text{Minimize} \quad \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} z_{ij},$$

subject to

$$x_i - x_j \le d_{ij} + Mz_{ij}, \quad 1 \le i < j \le n \qquad (3.2)$$

$$z_{ij} \in \{0, 1\} \text{ and } x \in C.$$

The implicit enumeration procedure will develop a binary tree to inspect all possible ordinal rankings; that is, all the elements of $C$. This will be accomplished by performing a dichotomy at each node that will

divide the current interval within which an $x_i$ may be contained into two disjoint subintervals, and then forcing $x_i$ to be in one subinterval down the "left" branch and in the other subinterval down the "right" branch. Generally, it will only be necessary to generate a small portion of the complete tree, as many solutions can be discarded as being suboptimal. Our algorithm will define rules for developing the tree, obtaining bounds and identifying an optimal solution.

It is noted that a branch-and-bound algorithm to solve (3.2) may be created which would dichotomize on $z_{ij}$, setting it to zero down one branch and to one down another. The difficulty with this type of procedure is that a nontrivial problem may arise to determine a feasible solution. It seems impossible to avoid a sub-enumerative process at certain nodes to either obtain a feasible solution or to verify that none exists. For this reason, we have chosen the partitioning procedure as previously described. In the case of dichotomizing on the $x_i$ variables, we will state a simple procedure for obtaining a feasible solution when one exists. While this solution may not be optimal for the candidate problem (see Geoffrion and Marsten [6] as a source for the terminology used to describe the tree), it does provide an upper bound on the objective value of all descendants of the node.

For any node (say the $p^{th}$) in the solution tree we define the following:

$L_i^p$ a lower bound on the value of $x_i$;

$U_i^p$ an upper bound on the value of $x_i$;

$LB^p$ a lower bound on the optimal objective value of any feasible completions of the tree emanating from node $p$;

$UB^p$ an upper bound on the optimal objective value of any feasible completions of the tree emanating from node $p$;

$F^p$ an index set containing the indices of all objects which, no matter what ranking is assigned in the interval $[L_i^p, U_i^p]$, the lower bound will be unaffected now and in all future descendants;

$R^p$ the best known ranking satisfying the restrictions imposed at node $p$;

$UB$ $\min_p [UB^p]$.

The values for $L_i^p$ and $U_i^p$ will be assigned by the branching process. The objective value associated with the current solution equals $UB^p$. The values for $LB^p$, $F^p$ and $R^p$ will be determined by the algorithm in a manner to be described.

We begin the description of the algorithm by specifying how the interval within which $x_i$ must be contained will be partitioned into smaller and smaller subintervals. This is performed in a manner to guarantee that

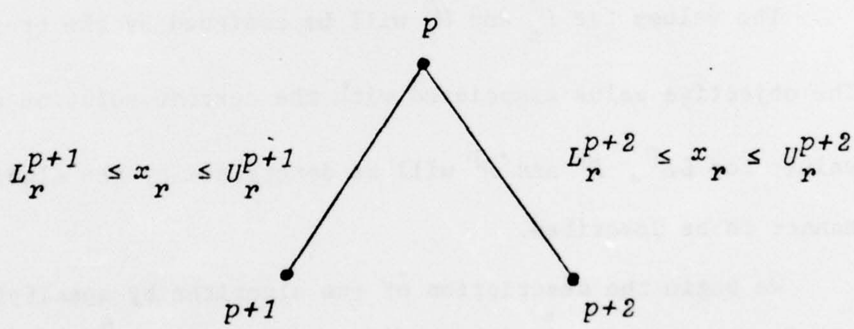
$$[L_i^p, U_i^p] \cap [L_k^p, U_k^p]$$

equals either (3.3)

$$[L_i^p, U_i^p], [L_k^p, U_k^p] \text{ or } \phi \text{ for all } i, k \text{ and } p.$$

That is, all the intervals are either disjoint or one is completely contained within the other. One way to maintain this condition is to begin with $L_i^o = 1$ and $U_i^o = n$, for all $i$ and then use the same partitioning sequence on every $x_i$. To demonstrate how (3.3) facilitates the implementation of our algorithm, we state and prove the following theorem:

<u>Theorem</u> Let $\overline{X}$ be an ordinal ranking which satisfies the interval constraints $L_i^p \leq x_i \leq U_i^p$, $i = 1, 2, \ldots, n$ at node $p$. Furthermore, let condition (3.3) be satisfied. Suppose two new candidate problems are created at nodes $p+1$ and $p+2$ as follows:




where $[L_r^{p+1}, U_r^{p+1}] \cup [L_r^{p+2}, U_r^{p+2}] = [L_r^p, U_r^p]$.

If this partitioning maintains (3.3) then any infeasibility which may arise can either be corrected by performing a pairwise interchange between the current value of $x_r (\overline{x}_r)$ and the current value of some $x_i$, or no feasible solution exists.

<u>Proof</u> Let $[L_r^{p+1},\ U_r^{p+1}]$ be the interval within which $x_r$ must lie in the new candidate problem. The intervals for all other variables remain unchanged; that is, $[L_i^{p+1},\ U_i^{p+1}] = [L_i^p,\ U_i^p]$, $i \neq r$. Clearly, if $L_r^{p+1} \leq \bar{x}_r \leq U_r^{p+1}$, then $\bar{X}$ is still feasible. The case of interest arises when $x_r = \bar{x}_r$ is not feasible. If $\bar{x}_r$ is not contained in $[L_r^{p+1},\ U_r^{p+1}]$ then we may search over all $x_k$ such that

$$L_r^{p+1} \leq \bar{x}_k \leq U_r^{p+1} \tag{3.4}$$

to see if
$$L_k^p \leq \bar{x}_r \leq U_k^p\ . \tag{3.5}$$

For any $k$ satisfying (3.4) and (3.5), a feasible solution at node $p+1$ is immediately obtained by placing $x_r = \bar{x}_k$, $x_k = \bar{x}_r$ and $x_i = \bar{x}_i$, $i \neq r, k$. On the other hand, if no $k$ exists which satisfies (3.4) and (3.5), then, to prove the theorem, it must be shown that no feasible solution exists at this node. If conditions (3.4) and (3.5) cannot be satisfied, then

$$[L_k^p,\ U_k^p] \cap [L_r^p,\ U_r^p] = [L_k^p,\ U_k^p] \tag{3.6}$$

is a proper subset of $[L_r^p,\ U_r^p]$. Also

$$[L_k^p, U_k^p] \cap \{[L_r^{p+1}, U_r^{p+1}] \cup [L_r^{p+2}, U_r^{p+2}]\}$$

$$= \{[L_k^p, U_k^p] \cap [L_r^{p+1}, U_r^{p+1}]\} \cup \{[L_k^p, U_k^p] \cap [L_r^{p+2}, U_r^{p+2}]\}.$$

Clearly, if

$$L_r^{p+1} \leq x_k \leq U_r^{p+1}$$

is feasible, then by conditions (3.3) and (3.6)

$$[L_k^p, U_k^p] \cap [L_r^{p+2}, U_r^{p+2}] = \phi.$$

This means that

$$[L_k^p, U_k^p] \cap [L_r^{p+1}, U_r^{p+1}] = [L_k^p, U_k^p]$$

and, therefore, $x_k$ cannot be assigned a value outside the interval $[L_r^{p+1}, U_r^{p+1}]$. Hence, all the projects currently assigned a ranking in this interval must remain in the interval and, in addition, to maintain feasibility the $r^{th}$ project must be assigned a ranking in this interval. This implies that there are more projects than there are rankings to assign, and the restricted problem at node $p+1$ has no feasible solution. □

By utilizing the results of the theorem, it is a simple matter to maintain a feasible solution. This solution provides us with an upper bound on the optimal objective value at that node. Before a step-by-step statement of the algorithm is given, a method for obtaining a lower bound on the optimal objective value will be discussed.

In this paper we will present easily implemented rules for determining $F^p$ and $LB^p$. The following is a step-by-step statement of our algorithm. The rules are based on the following observations.

(i) If we have any hope of satisfying the constraint

$$x_i - x_j \leq d_{ij} ,$$

then we must have

$$L_i - U_j \leq d_{ij} .$$

Thus, by counting the number of pairs $(i,j)$ with $i < j$ such that $L_i^p - U_j^p \leq d_{ij}$, a lower bound on the objective value of any descendants of node $p$ can be obtained.

(ii) The constraint $x_i - x_j \leq d_{ij}$ will always be satisfied by feasible solutions to descendants of node $p$ whenever $U_i^p - L_j^p \leq d_{ij}$. Therefore, an object $k$ with

$$U_k^p - L_j^p \leq d_{kj} \text{ or } L_k^p - U_j^p > d_{kj} \quad \text{for all } j > k$$

$$U_j^p - L_k^p \leq d_{ik} \text{ or } L_i^p - U_k^p > d_{ik} \quad \text{for all } i < k,$$

can be placed anywhere in its allowable interval without affecting the objective value of any descendants from node $p$. Hence, $k$ can be placed in the previously defined index set $F^p$.

It is possible to obtain stronger bounds but it appears that the computations required to obtain these bounds would not justify the savings achieved by producing a smaller solution tree.

The Algorithm

STEP 1. Set $p = t = LB^o = 0$. Set $R^o = G$.

STEP 2. Calculate the current objective value with $R^o$; that is, how many constraints are violated with $z_{ij} = 0$ for every $(i, j)$. Set $UB$ equal to this objective value. Set $L_i^o = 1$, $i = 1, 2, \ldots, n$ and $U_i^o = n$, $i = 1, 2, \ldots, n$.

STEP 3. $F^o = \{k : |d_{kj}| \geq n - 1, \text{ for all } j \neq k\}$.

STEP 4. For $1 \leq i < j \leq n$, $LB^o$ equals the cardinality of $V^o$, where

$$V^o = \{(i, j) \mid L_i - U_j > d_{ij} \text{ for } i < j\}.$$

STEP 5. Let $LB^p = \min LB^r$, $r = 0, 1, \ldots, t$. If $LB^p = UB$ go to step 10; otherwise, go to step 6.

STEP 6. Let $k$ equal the index of the variable to be further restricted.

$$U_k^p - L_k^p = \text{maximum } U_i^p - L_i^p,\ i \notin F.$$

Ties are broken by choosing the smallest index satisfying the above criterion.

STEP 7. Create two new nodes $t+1$ and $t+2$. At node $t+1$ assign a new upper bound on the variable $x_k$ as follows.

$$U_k^{t+1} = L_k^p + [(U_k^p - L_k^p)/2]$$

where [ ] indicates greatest integer less than or equal to the number in the brackets.

STEP 7. cont'd. At node $t+2$ assign a new lower bound on the variable $x_k$ as follows:

$$L_k^{t+2} = U_k^{t+1} + 1.$$

<u>STEP 8.</u> Check to see if $k$ can be placed in either $F^{t+1}$ or $F^{t+2}$. The index $k$ can be placed in $F^{\ell}$ whenever

$$U_k^{\ell} - L_j^{\ell} \le d_{kj} \text{ or } L_k^{\ell} - U_j^{\ell} > d_{kj} \text{ for all } k < j$$

and

$$U_i^{\ell} - L_k^{\ell} \le d_{ik} \text{ or } L_i^{\ell} - U_k^{\ell} > d_{ik} \text{ for } k > i$$

where

$$\ell = t+1 \text{ or } \ell = t+2$$

$$LB^{t+2} = LB^{t+1} = LB^{p}, \quad V^{t+2} = V^{t+1} = V^{p}.$$

Also, if

$$L_k^{t+1} - U_j^{t+1} > d_{kj} \text{ for } k < j$$

or

$$L_i^{t+1} - U_k^{t+1} > d_{ik}$$

add the index pair $(k, j)$ or $(i, k)$ to $V^{t+1}$ if the pair is not already present; furthermore, if the pair is not already in $V^{p}$, $LB^{t+1} = LB^{t+1} + 1$. Similarly, if

$$L_k^{t+2} - U_j^{t+2} > d_{kj}, \; k < j$$

or

$$L_i^{t+2} - U_k^{t+2} > d_{ik} \; ,$$

add the index pair $(k, j)$ or $(i, k)$ to $V^{t+2}$ if the pair is not already present. If the pair is not present $LB^{t+2} = LB^{t+2} + 1$. Set $LB^p = n^2$.

STEP 9. For the new nodes obtain a new feasible solution $R^{t+1}$ and $R^{t+2}$. Notice that at one of the nodes ($t+1$ or $t+2$) $R^p$ will be feasible. A feasible solution at the other node can either easily be obtained (because our intervals bounds are non-overlapping) by rearranging the order or no feasible solution exists at that node. If no feasible solution exists set the lower bound equal to $n^2$. Define a new objective value from $R^{t+1}$ or $R^{t+2}$ (the one that is different). Go to step 5.

STEP 10. Terminate-- the solution yielding $UB$ is optimal.

## Weights

In applications such as those discussed previously the introduction of weights is essential. It may be necessary to consider two different forms of weights.

Firstly, the $\{Q^\ell\}_{\ell=1}^m$ may not all be of equal importance. If we introduce relative weights $w^\ell$ then

$$Y = \sum_{\ell=1}^{m} w^\ell Q^\ell$$

and

$$d_{ij} = \sum_{\ell=1}^{m} w^\ell (q_i^\ell - q_j^\ell).$$

Without loss of generality we assume the weight attached to $X$ is $w^{m+1} \equiv 1$. Weighted rankings, thus, present no computational problems.

A second consideration which must be made with regard to weights has to do with the relative importance of pairwise preferences. Relative to the desired ranking $G$, it may be, for example, that the preference of project 1 to project 2 is greater than that of project 2 to project 3. If $w_{ij}$ denotes the relative importance of the preference for project $i$ over $j$ then the objective function of (3.1) (and (3.2)) becomes

$$\text{minimize} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} w_{ij} z_{ij} .$$

With this more general formulation, appropriate modifications of the algorithm are in order. Specifically, in:

STEP 4; $$LB^{o} = \sum_{(ij) \in V^{o}} w_{ij}$$

STEP 8; Instead of $$LB^{t+1} = LB^{t+1} + 1$$

we have
$$LB^{t+1} = LB^{t+1} + w_{kj} \text{ (or } w_{ik}\text{)}.$$

A similar modification must be made to $LB^{t+2}$ if necessary.

In the following section we present a detailed example to illustrate the algorithm.

4. Sample Problem

**The fifth member of a five-member committee wishes to provide an ordinal ranking for six objects that will provide a consensus ranking as close as possible to $G = (1, 2, . . . , 6)$. It is known that the other committee members have provided the following rankings.**

$$q^1 = \begin{pmatrix} 3 \\ 2 \\ 5 \\ 1 \\ 6 \\ 4 \end{pmatrix} \qquad q^2 = \begin{pmatrix} 1 \\ 3 \\ 6 \\ 2 \\ 4 \\ 5 \end{pmatrix}$$

$$q^3 = \begin{pmatrix} 3 \\ 2 \\ 5 \\ 1 \\ 4 \\ 6 \end{pmatrix} \qquad q^4 = \begin{pmatrix} 3 \\ 1 \\ 6 \\ 2 \\ 4 \\ 5 \end{pmatrix}$$

**Therefore,**

$$y = \begin{pmatrix} 10 \\ 8 \\ 22 \\ 6 \\ 18 \\ 20 \end{pmatrix}$$

We assume without loss of generality that all weights equal *1*.

Table 1 gives the pertinent information generated by the algorithm during the branch-and-bound process. In this example the algorithm always branched to one of the newly created nodes and optimality was verified at node *17*. We began with a lower bound of *1* because the desired ordinal relationship between objects *3* and *4* cannot be satisfied.

TABLE 1

| Node | 0 | 1 | 4 | 5 | 8 |
|---|---|---|---|---|---|
| Constraint added | - | $1 \le x_1 \le 3$ | $4 \le x_2 \le 6$ | $1 \le x_3 \le 3$ | $4 \le x_4 \le 6$ |
| Upper bound | 5 | 5 | 4 | 4 | 4 |
| Lower bound | 1 | 1 | 1 | 1 | 1 |
| Solution $R^p$ | 1 | 1 | 1 | 1 | 1 |
| | 2 | 2 | 4 | 4 | 4 |
| | 3 | 3 | 3 | 3 | 3 |
| | 4 | 4 | 2 | 2 | 6 |
| | 5 | 5 | 5 | 5 | 5 |
| | 6 | 6 | 6 | 6 | 2 |

| Node | 10 | 11 | 14 | 15 | 17 |
|---|---|---|---|---|---|
| Constraint added | $4 \le x_5 \le 6$ | $1 \le x_6 \le 3$ | $2 \le x_1 \le 3$ | $4 \le x_2 \le 4$ | $1 \le x_3 \le 1$ |
| Upper bound | 4 | 4 | 4 | 4 | 1 |
| Lower bound | 1 | 1 | 1 | 1 | 1 |
| Solution $R^p$ | 1 | 1 | 2 | 2 | 2 |
| | 4 | 4 | 4 | 4 | 4 |
| | 3 | 3 | 3 | 3 | 1 |
| | 6 | 6 | 6 | 6 | 6 |
| | 5 | 5 | 5 | 5 | 5 |
| | 2 | 2 | 1 | 1 | 3 |

Graphically, the solution tree appears as drawn in Figure 1. Nodes which were fathomed before the algorithm branched to them (that is, an upper bound calculated and further partitioning considered) are indicated with a slash.

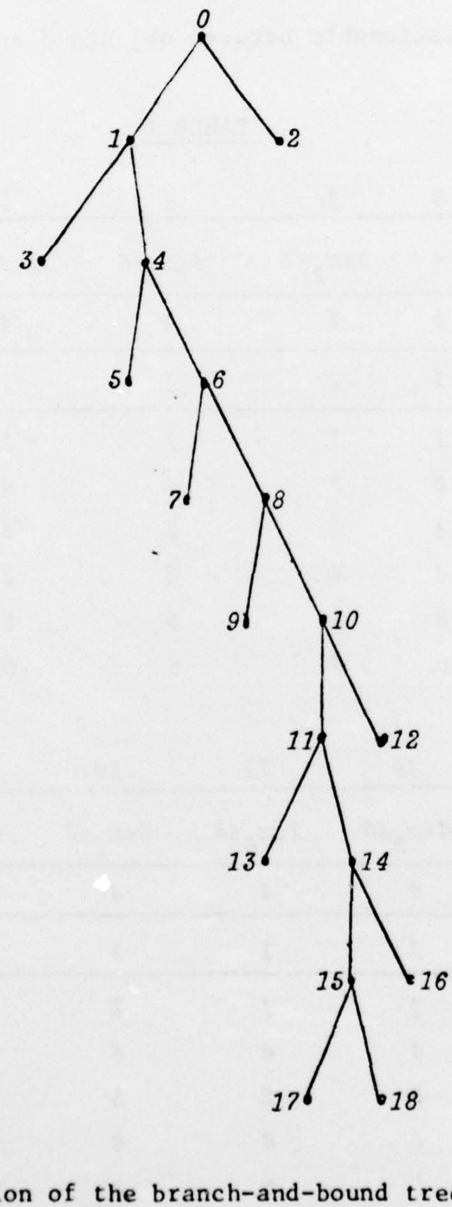



**Figure 1.** Illustration of the branch-and-bound tree created during the solution of the sample problem.

Clearly, a large number of branches may be required for the solution of a large problem. However, the cardinality of the index sets $V^p$ and $F^p$ would be expected to expand with the size of the problem. The strength in our algorithm lies in the ease with which a feasible solution can be obtained at any node.

As is the case with almost any algorithm, there are an infinite number of ways to modify our algorithm and actually implement it. For example, it may be of benefit to attempt the improvement of the current solution with pairwise interchanges, or update $V^p$ occasionally (say, every 10 branches) rather than at every node. We will not elaborate on any such modifications here.

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13. ABSTRACT

This paper investigates a class of committee ranking problems. A given member wishes to select an ordinal ranking of a set of objects which, when combined with the rankings provided by the other members, yields a consensus ranking that is as close as possible to some desired set of preferences. Areas of potential application such as defence construction and highway planning are discussed. An algorithm for determining the best ranking together with an illustrative example is given.

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